Mathematical Problems of Computer Science 63, 7–13, 2025.
doi:10.51408/1963-0127

UDC 529.6

Full-Reference Image Quality Assessment Procedure Based on Rice Distribution Model

David G. Asatryan[1,2], Liana K. Andreasyan[3] and Grigor S. Sazhumyan[2]

[1]Institute for Informatics and Automation Problems of NAS RA, Yerevan, Armenia
[2]Russian-Armenian University, Yerevan, Armenia
[3]National Polytechnic University of Armenia, Yerevan, Armenia
e-mail: dasat@iiap.sci.am, landreasyan@yahoo.com, grigorsazhumyan@gmail.com

Abstract

The problem of full-reference image quality assessment is considered based on the application of the mathematical model of the Rice distribution. The gradient field of an image is adequately described by the Weibull distribution, which allows one to effectively analyze image properties, evaluate their similarity, classify them by quality, etc. In this paper, an attempt is made to solve similar problems using the above-mentioned model, relying, in particular, on additional properties of the Rice distribution associated with the normal approximation of the latter. It is shown that the structural similarity measure used in different problems is also applicable to the case of the Rice gradient field model. In particular, images from the TID2013 database are experimentally studied. The modeling results obtained from both the Weibull and Rice distribution models were compared using the mean square and structural similarity measures, as well as the Mean Opinion Score (MOS) values. It is shown that the types of distortions in these indicators are in complete agreement, while for some other types, the Rice distribution model shows better results.

Keywords: Gradient magnitude, Weibull distribution, Rice distribution, Parameter estimation, Image similarity, MOS.

Article info: Received 29 January 2025; sent for review 6 Febuary 2025; accepted 28 April 2025.

1. Introduction

Creating effective quality assessment methods is one of the most popular and applied tasks in the field of image processing. The existing quality assessment methods are divided into two classes: Full-Reference and No-Reference methods. The Full-Reference method assumes the presence of an initial reference image and a set of test images that differ from the reference due to the impact of certain distorting factors on the reference. In this case, the task of quality assessment consists of comparing the test and reference images using a pre-selected criterion.

No-reference methods do not assume the presence of a standard and the quality of the tested image assessing using only the internal properties of the image. The literature devoted to methods for assessing image quality is quite extensive. Of particular interest are methods that use the properties of the human visual system (HVS), since the "final judge" of quality assessment is a person. These methods often include Mean Opinion Score (MOS) values obtained by experts for images pre-distorted by different methods, which allows for checking the ability of the tested method to assess image quality. For this purpose, researchers have created extensive image databases accompanied by MOS assessments. Brief descriptions and links to dozens of such databases are given in [1].

Previously, we proposed a method for assessing image quality based on statistical analysis of the gradient field of an image [2]-[3]. In this case, the Weibull distribution model was adopted to describe the set of gradient magnitudes. It is shown that this model allows for solving many relevant applied problems. Thus, in [4], a Full-Reference algorithm for assessing image quality is proposed using the TID2013 database of distorted samples [5], for which MOS values are also given. The latter circumstance allows for assessing the quality of both the tested image and the applied testing algorithm.

This paper attempted to supplement the proposed approach using other gradient field models. The proposed method is based on the Rice distribution, which, unlike the Weibull distribution, converges to a normal distribution with appropriate parameter values, thus creating additional opportunities for adequate analysis and assessment of image quality.

The paper considers the following tasks:

- Modeling Weibull and Rice distributions with the ability to estimate parameters using various methods.
- Calculating the similarity of the original test image with distorted samples based on the proximity of the parameter values of the original and distorted samples. Comparison and analysis of the results obtained for both distribution models.
- Comparison of similarity values with the corresponding MOS values, comparing and analyzing the existing discrepancies, and developing appropriate recommendations.

2. Research Methodology

The methodology involves modeling the Weibull and Rice distributions, calculating the magnitudes of the gradients of the tested images, and estimating the parameters of both distributions based on these data. In this case, the gradients are estimated using the Sobel operator. The parameters of the Weibull distribution are estimated using the method of moments [3], and the maximum likelihood method is used to estimate the parameters of the Rice distribution [6]. The experiments were conducted on images from the TID2013 database, which also contains other auxiliary information. The MOS values, PSNR and W^2 image similarity measures were used to compare the results.

The probability density function of the two-parameter Weibull distribution is defined by the formula

$$f(x;\lambda,\eta)=\frac{\eta}{\lambda}\left(\frac{x}{\lambda}\right)^{\eta-1}\exp\left[-\left(\frac{x}{\lambda}\right)^{\eta}\right],\ x\geq 0,$$

where $\eta > 0$ - Shape parameter, and $\lambda > 0$ - Scale parameter.

The Rice distribution density is

$$f(x|\nu,\sigma) = \frac{x}{\sigma^2} exp\left(\frac{-(x^2+\nu^2)}{2\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right),$$

where $I_0(z)$- modified Bessel function of the first kind of zero order.

Unlike the Weibull distribution, in the literature, it is customary to define the shape and scale parameters in the Rice case as the following functions of the initial parameters ν and σ: *Shape parameter* $K = \frac{\nu^2}{2\sigma^2}$, and *Scale parameter* $\Omega = \nu^2 + 2\sigma^2$. With the values of these quantities, we can estimate the initial parameters of the Rice distribution using the formulas

$$\nu = \sqrt{\frac{K\Omega}{K+1}}, \qquad \sigma = \sqrt{\frac{\Omega}{2(K+1)}}.$$

It should be noted that in the field of signal processing theory and technology [7], an important characteristic is the signal-to-noise ratio, which is determined by the expression $\xi = \nu/\sigma$. It is known that $\xi\to\infty$ the Rice distribution tends to a normal distribution with parameters ν and σ. Moreover, for $\xi \geq 3$, this approximation is quite acceptable. This means that under this condition, we have $K \geq 4.5$, and the estimation of the parameters of the Rice distribution can be performed by traditional statistical methods.

The similarity (closeness) of two images can be estimated by the degree of closeness of the corresponding empirical Weibull or Rice distributions constructed from the set of magnitudes of the gradients of the compared images. However, instead of nonparametric statistical criteria of goodness of fit, we use a less accurate but simple measure [2], based on the closeness of the parameter estimates of the of the distributions under study themselves according to the formula

$$W^2 = \frac{\min(\eta_1,\eta_2)\min(\lambda_1,\lambda_2)}{\max(\eta_1,\eta_2)\max(\lambda_1,\lambda_2)} \quad 0 < W^2 \leq 1.$$

The resulting image similarity scores were then compared with the MOS scores using the Spearman correlation coefficient. This measure is often conveniently assessed by visual analysis, classifying their absolute values as equal to or less than one.

We have repeatedly and successfully applied the described method to various problems [3]. In particular, in [4], by analyzing the database data, the types of distortions for which the estimates of the Weibull distribution parameters belonged to one or another class from those described above were identified. In the present work, a similar analysis was carried out concerning the Rice distribution data.

3. Results of Modeling

Modeling was performed on all 3000 images of the database [5] simultaneously using both models. By analyzing the modeling results, the types of distortion identified for which the Weibull model leads to high or low values of the W^2 correlation with MOS. The corresponding values obtained using the Rician model are also recorded. It turned out those types of distortions, and chaotic behaviour of W^2 estimates were observed using the Weibull model, while the situation is much better using the Rician model. Of course, there may also be types of distortions for which the behaviour of these estimates completely coincides. Examples of this kind are given below.

Example 1. Matching indicators. Let us consider the images I01_01 and I01_04 of the base. Table 1 shows the calculation results.

Table 1. Comparative results for images I01 and I04

I01_01	PSNR	W^2 Weibull	W^2 Rician	MOS	i01_04	PSNR	W^2 Weibull	W^2 Rician	MOS
i01_01_1	36.50	0.92	0.92	5.51429	i01_04_1	30.14	0.69	0.69	5.76190
i01_01_2	33.56	0.87	0.87	5.56757	i01_04_2	29.26	0.55	0.55	5.47619
i01_01_3	30.48	0.79	0.78	4.94444	i01_04_3	27.85	0.40	0.38	4.92857
i01_01_4	27.51	0.68	0.66	4.37838	i01_04_4	25.95	0.27	0.23	4.26829
i01_01_5	24.50	0.56	0.51	3.86486	i01_04_5	23.61	0.18	0.13	4.00000

Visual analysis of the data in Table 1 shows that the nature of the change in the values of the considered indicators for these images is generally the same. First, we note the monotonic decrease in PSNR with an increase in the degree of applied distortion, which is a serious argument for using this indicator in the absence of MOS-type data. In this case, PSNR can also be used to assess the quality of the experiment to create MOS data. We also note the practical coincidence of the W^2 values for the considered images and the Weibull and Rice distributions, despite some deviations in the MOS series. However, as shown in [4], these patterns are not always observed, so several similar examples with appropriate comments are given below.

Table 2 compares the calculation results for images i05 and i07, subjected to the same type of distortion (Contrast change). As can be seen, PSNR decreases monotonically in both cases, and W^2 with the Rice distribution also decreases monotonically in the case of image i05_17. However, deviations from monotony are observed for i0._17.

Table 2. Comparison of similarity scores with MOS scores

I05_17	PSNR	W^2 Weibull	W^2 Rician	MOS	i07_17	PSNR	W^2 Weibull	W^2 Rician	MOS
1	33.51	0.9	0.79	5.3	1	35,97	0.9	0.8	5.54545
2	28.92	0.85	0.74	6.82927	2	30.28	0.84	0.77	6.4
3	25.53	0.75	0.55	4.025	3	28.06	0.75	0.55	4.40476
4	22.81	0.77	0.53	6.56098	4	23.60	0.72	0.6	6.72727
5	19.47	0.5	0.24	2.8	5	22.00	0.5	0.25	3.34091

The MOS values do not decrease monotonically, as expected by the meaning of the experiments, but the nature of the changes is similar for both images. This effect can be explained by the peculiarities of the human visual system (HVS) that inadequately react to changes in image contrast in one direction or another.

Thus, the results of Table 2 indicate some advantages of using the Rice distribution when assessing image quality.

It is interesting to compare the calculation results for the same image with different types of distortion. Table 3 shows the data for the i04 image with changes in brightness (Mean shift (intensity shift)) and contrast (Contrast change). In this case, deviations from the monotony of the similarity indices W^2 with the Weibull model and MOS were observed, while W^2 with the Rician model decreased monotonically, corresponding to the meaning of the experiment on creating the TID2013 database.

Table 3. Comparison of ratings for different types of image distortion

I04_16	PSNR	W^2 Weibull	W^2 Rician	MOS	i04_17	PSNR	W^2 Weibull	W^2 Rician	MOS
1	33.33	0.99	0.84	6.28571	1	30.51	0.9	0.84	6
2	24.63	1	0.68	6.64286	2	28.79	0.81	0.68	6.66667
3	24.52	0.98	0.59	6.09524	3	27.62	0.75	0.59	4.78049
4	17.57	0.99	0.50	5.66667	4	24.47	0.69	0.50	7.21429
5	17.94	0.79	0.26	5.15385	5	23.13	0.50	0.26	3.925

Similar results were obtained for images i21, i24 and several others from the same database with the same types of distortions.

4. Conclusions

The problem of Full-Reference image quality assessment is considered based on applying the mathematical model of the Rice distribution. The previously proposed technique is based on the application of the Weibull distribution model and the measures of mean square and structural similarity of images. In this paper, the properties of images from the TID2013 database are experimentally investigated, evaluating and comparing their similarity indices according to the Weibull and Rice models, as well as the MOS index. This shows that the applied measure of structural similarity is also applicable to the case of the Rice gradient field model. It is also shown that for types of distortion, these indices are in complete agreement, while for some other types; the Rice distribution model shows better results.

References

[1] G. Zhai and X. Min, “Perceptual image quality assessment: a survey”, *Sci. China Inf. Sci.*, vol. 63, no.11, 211301, 2020. doi:10.1007/s11432-019-2757-1

[2] D. Asatryan and K. Egiazarian, “Quality assessment measure based on image structural properties”, *2009 International Workshop on Local and Non-Local Approximation in Image Processing*, Tuusula, Finland, pp. 70-73, 2009. doi:10.1109/lnla.2009.5278400

[3] D. Asatryan, “Gradient-based technique for image structural analysis and applications”, *Computer Optics*, vol.43, no. 2,pp.245-250, 2019. doi: 10.18287/2412-6179-2019-43-2-245-250.

[4] D. Asatryan, M. Haroutunian, G. Sazhumyan and G. Hakobyan, “Procedure for analyzing the quality, structure and subjective rating of distorted images by the Full-Reference technique”, *Intern. Scientific Journal Mathematical Modeling*, 2022, vol. 6, no.4, pp. 100-102, 2022.

[5] N. Ponomarenko, L. Jin, O. Ieremeiev, V. Lukin, K. Egiazarian, J. Astola, B. Vozel, K. Chehdi, M. Carli, F. Battisti and C.-C. Jay Kuo, “Image database TID2013: Peculiarities, results and perspectives”, *Signal Processing: Image Communication*, vol. 30, pp. 57-77, 2015.

[6] T.Yakovleva, “Peculiarities of the rice statistical distribution: mathematical substantiation”, *Applied and Computational Mathematics*, Science Publishing Group, vol.7, no. 4, pp. 188-196, 2018. doi: 10.11648/j.acm.20180704.12.
[7] Б. Левин, “Теоретические основы статистической радиотехники”*, М.: Радио и связь*, 656 с., 1989.

Ռայսի բաշխման մոդելի հիման վրա ստուգանմուշի հետ համեմատման մեթոդով պատկերի որակի գնահատման ընթացակարգ

Դավիթ Գ. Ասատրյան[1,2], Լիանա Կ. Անդրեասյան[3] և Գրիգոր Ս. Սաժումյան[2]

[1]ՀՀ ԳԱԱ Ինֆորմատիկայի և ավտոմոտացման պրոբլեմների ինստիտուտ, Երևան, Հայաստան
[2]Հայ-Ռուսական համալսարան, Երևան, Հայաստան
[3]Հայաստանի ազգային պոլիտեխնիկական համալսարան, Երևան, Հայաստան
e-mail: dasat@iiap.sci.am, landreasyan@yahoo.com, grigorsazhumyan@gmail.com

Ամփոփում

Դիտարկվում է ստուգանմուշի հետ համեմատման մեթոդով պատկերի որակի գնահատման խնդիրը Ռայսի բաշխման մաթեմատիկական մոդելի կիրառման հիման վրա: Հայտնի է, որ պատկերի գրադիենտային դաշտը համարժեքորեն նկարագրվում է Վեյբուլի բաշխմամբ, ինչը թույլ է տալիս արդյունավետորեն վերլուծել պատկերների հատկությունները, գնահատել դրանց նմանությունը, դասակարգել դրանք ըստ որակի և այլն: Այս հոդվածում փորձ է արվել լուծել նմանատիպ խնդիրներ՝ օգտագործելով վերը նշված մոդելը՝ հենվելով, մասնավորապես, Ռայսի բաշխման լրացուցիչ հատկությունների վրա՝ կապված վերջինիս նորմալ մոտարկման հետ: Ցույց է տրվել, որ տարբեր խնդիրներում օգտագործվող կառուցվածքային նմանության չափանիշը կիրառելի է նաև Ռայսի մոդելի դեպքում: Մասնավորապես, փորձնականորեն ուսումնասիրվել են TID2013 տվյալների բազայի պատկերները: Վեյբուլի և Ռայսի բաշխման մոդելների միջոցով ստացված մոդելավորման արդյունքները համեմատվել են՝ օգտագործելով միջին քառակուսային և կառուցվածքային նմանության չափանիշները, ինչպես նաև՝ փորձագիտական կարծիքի միջին գնահատականի արժեքները (MOS): Ցույց է տրված, որ աղավաղումների տեսակների այս ցուցանիշները լիովին համընկնում են, մինչդեռ որոշ այլ տեսակների համար Ռայսի բաշխման մոդելը ցույց է տալիս ավելի լավ արդյունքներ:

Բանալի բառեր՝ գրադիենտային մագնիտուտ, Վեյբուլի բաշխում, Ռայսի բաշխում, պարամետրերի գնահատում, պատկերների նմանություն, կարծիքի միջին գնահատական (MOS):

Процедура оценивания качества изображения методом сравнения с эталоном на основе модели распределения Райса

Давид Г. Асатрян[1,2], Лиана К. Андреасян[3] и Григор С. Сажумян[2]

[1]Институт проблем информатики и автоматизации НАН РА, Ереван, Армения
[2]Российско-Армянский университет, Ереван, Армения
[3]Национальный политехнический университет Армении, Ереван, Армения
e-mail: dasat@iiap.sci.am, landreasyan@yahoo.com, grigorsazhumyan@gmail.com

Аннотация

Рассматривается задача оценивания качества изображения методом сравнения с эталоном, основанная на применении математической модели распределения Райса. Известно, что градиентное поле изображения достаточно адекватно описывается распределением Вейбулла, что позволяет эффективно анализировать свойства изображений, оценивать их сходство, классифицировать по качеству и др. В данной работе сделана попытка решать аналогичные задачи по упомянутой модели, рассчитывая, в частности, на дополнительные свойства распределения Райса, связанные с нормальным приближением последнего. Показано, что применяемая в разных задачах мера структурного сходства применима и в случае райсовской модели градиентного поля. В частности, экспериментально исследованы изображения из базы данных TID2013. Сопоставлены результаты моделирования, полученные по моделям распределений Вейбулла и Райса, используя меры среднеквадратического и структурного сходства, а также значения экспертных оценок (MOS). Показано, что для определённых типов искажений эти показатели находятся в полном согласии, в то время как для некоторых других типов модель распределения Райса показывает лучшие результаты.

Ключевые слова: магнитуда градиента, распределение Вейбулла, распределение Райса, оценивание параметров, сходство изображений, MOS.